DUKE UNIVERSITY, FUQUA SCHOOL OF BUSINESS
ACCOUNTG 512F: FUNDAMENTALS OF FINANCIAL ANALYSIS

Note on Cost of Capital

For the course, you should concentrate on the CAPM and the weighted average cost of capital. The text about the levering and unlevering of betas, as well as the 3-factor model, is for your information only. Note, however, that if you interview within the financial industry, they may expect you to have an idea about these things as well. Feel free to talk to me off-line if you need more info about anything cost-of-capital related. /PO

I. Market Efficiency

The theory of efficient capital markets holds that the purchase or sale of a security is a zero net present value transaction. That is, in a competitive market, prices are set in a way that reflects the intrinsic values of securities. Theories about the intrinsic values of securities invariably describe the value of a security as a function of the risks and the returns of that security. In particular, the risk-return tradeoff is the fundamental idea behind the discounted dividend model, the discounted free cash flow model, the discounted abnormal earnings model and the discounted economic value added model. Each of these models takes a payment stream (the return) and calculates its present value using a discount rate that consistently reflects the uncertainty (or risk) of those payments. The risk measures for these models include the cost of equity, the cost of the unlevered firm, the weighted average cost of capital, etc.

In order for market prices to reflect intrinsic values, information must be readily available to those investors who are critical in setting market prices. (That is, we do not require that all investors have access to such information, or even that all publicly available information is used by all investors. We require only that the marginal investor – the one(s) we believe set(s) security prices – has all relevant information and is able to properly interpret this information.) Stock prices become efficient in at least two ways. The first is through the activities of market participants, such as securities analysts and investors, performing fundamental analysis; the second is through the activities of investors applying technical screens. Fundamental analysis attempts to seek out information about a company, its industry, etc. Technical screens seek out, and trade based on, information in the patterns of prior returns, trading volume and other market metrics.

Sometimes we speak of various levels of market efficiency, depending on how strict a view of efficiency we wish to take:

- Weak market efficiency: Stock prices reflect all information in prior stock prices.
- Semi-strong market efficiency: Stock prices reflect all publicly available information.
- Strong market efficiency: Stock prices reflect all information (including insider information).

In general, when people assert market efficiency, they are referring to semi-strong market efficiency.

There is much evidence (albeit with mixed results) on whether stock markets in the U.S. and elsewhere are efficient. Given the high liquidity of U.S. markets and the extensive disclosure and accounting requirements of U.S. GAAP, it is generally believed that the U.S. has among the most efficient capital markets in the world. Evidence supporting market efficiency includes the well-

documented finding that security prices follow a random walk. Evidence refuting market efficiency includes studies documenting abnormal returns to contrarian and momentum strategies based on prior stock returns.

II. Risk and Return

If stock prices reflect the intrinsic value of firms, then it should be the case that firms that invest in riskier projects have higher *expected* returns. The expected premium is the compensation to investors for investing in a firm with more uncertain payoffs. For common equity securities, investors must be compensated for the use of their money (i.e., the pure interest rate value of the funds), market risk (the risk of investing in equity securities, as opposed to say fixed payment debt securities), and firm-specific risk (the incremental risk that a firm chooses when it selects more, or less, risky projects that makes its stock more, or less, exposed to the state of the economy). The risk-free rate, or the pure interest rate value, is typically proxied for by the yield on government obligations (short-term Treasury bills or medium-/long-tem Treasury bonds – the length depends on the assumed investment horizon). The difference between the total expected return from a risky investment and the risk-free rate (so the combination of market risk and firm-specific risk) is referred to as the *risk premium*. This can be formalized in the Capital Asset Pricing Model (CAPM), which tells us what the relation between risk and expected return "should" look like for all assets.

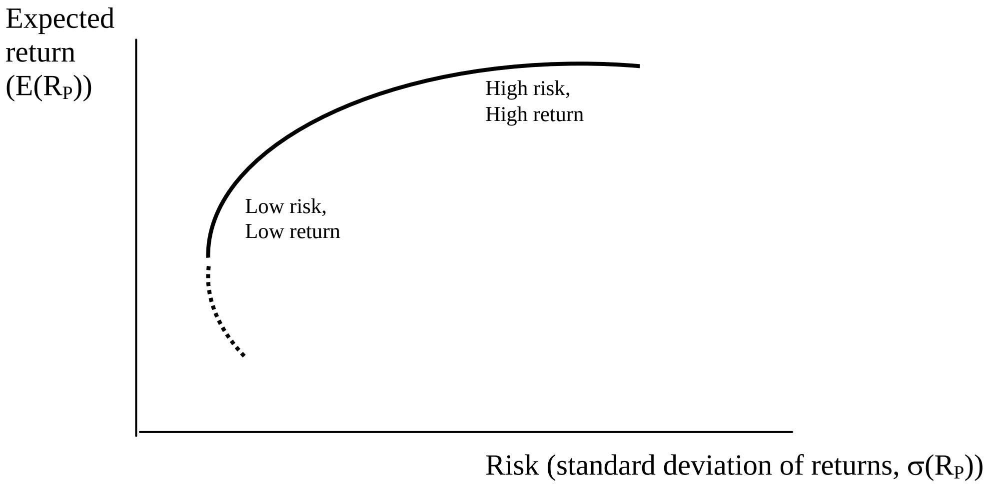
III. The Capital Asset Pricing Model (CAPM)

The most common form of the CAPM, the Sharpe-Lintner CAPM, describes the relation between Asset j's expected return, its risk, and the amount of the risk premium ("Asset j" can be an individual security or a portfolio). The model states that the expected return for any asset equals the risk-free rate plus the asset's beta times the expected market risk premium:

$$E(R_j) = R_F + \beta_j [E(R_M) - R_F] \tag{1}$$

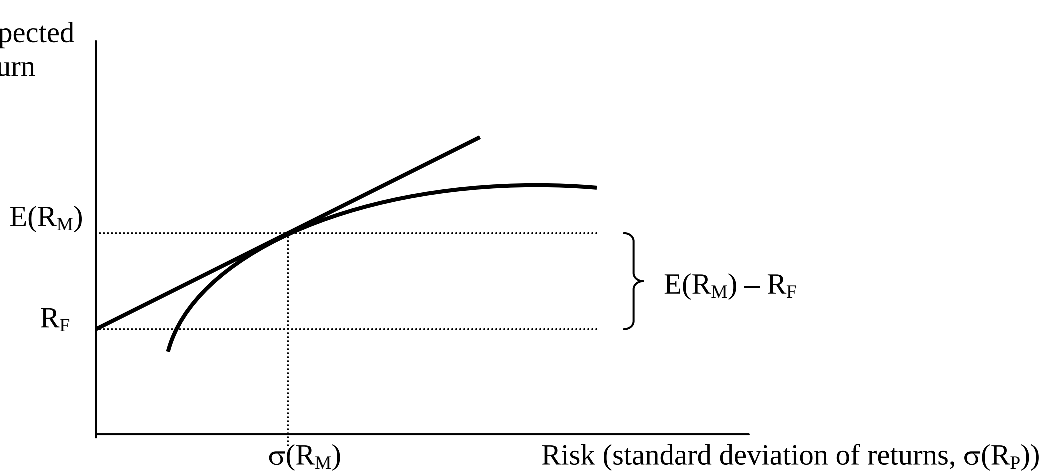
$E(R_j)$ = expected return on asset j
M = market portfolio
F = risk free asset
$E(R_M)$ = expected return on the market portfolio
R_F = risk-free rate

The derivation of the CAPM builds largely on mathematics (portfolio algebra), but also on a couple of assumptions. Specifically, it is assumed that investors dislike risk (in the CAPM captured by the standard deviation of returns or simply volatility), but that they like returns. Hence, for any given expected return investors try to minimize risk and for any given risk level, they will try to maximize their expected return. If we give investors access to multiple assets – so they can form a portfolio of assets – they will attempt to combine those assets so as to minimize the combined risk for a given level of portfolio return. This gives rise to the following relation between expected portfolio return and standard deviation, the "efficient frontier:"



The entire curve represents portfolios where risk is minimized. The solid part of the curve is the *efficient frontier*, which represents the maximum level of return for a given level of risk. No rational investor would choose a portfolio that was below the efficient frontier (because one could achieve a higher level of return for the same amount of risk). Conversely, no one *can* choose a portfolio above the efficient frontier (because such portfolios are simply not feasible in the given investment universe).

If we now add a new 'asset,' risk-free borrowing and lending, we get one more point in our graph: an asset return with a fixed return and therefore zero risk, the risk-free rate, R_F. This enables us to get above the old efficient frontier. Specifically, we can now *combine* the risk-free asset with a portfolio of risky assets to reach a higher level of expected return for any risk we may be willing to take. You can see from the graph that regardless of your risk preference (that is, how much risk you are willing to bear) you would always combine the risk free asset with one particular portfolio of risky assets, the tangent portfolio. Since all rational investors would do the same, only one portfolio would be held by all investors (in different proportions). This tangent portfolio is the market portfolio. Why? – Because in equilibrium all stocks have to be held by someone, and since all investors hold the same tangent portfolio, that tangent portfolio must include all stocks, i.e., it must be the market portfolio (and to be precise, the value-weighted market portfolio).



We now "know" that all investors choose the market portfolio, and that the market portfolio is efficient (recall that it lies on the efficient frontier). This allows us to make use of the general mathematics of efficient portfolios to specify the relation between the expected return of security j and its risk *within* the market portfolio. This is the CAPM formula:[1]

$$E(R_j) = R_F + \beta_j [E(R_M) - R_F] \tag{2}$$

where β_j = standardized sensitivity of security j to market-wide movements
= covariance of (R_j, R_M) / variance of (R_M).

One can, of course, criticize the CAPM. And people do. For example, the model says that all investors hold the market portfolio in combination with borrowing/lending. We do not observe this. Furthermore, borrowing and lending is never completely risk-free in real life. That said, all models are stylized simplifications of reality and therefore false to some extent, and a model's importance really lies in its usefulness. Because of its theory-based and yet intuitive derivation and its easy computation, the CAPM has long been the most commonly used model for determining discount rates. We will next cover how one implements the CAPM and its extension, the so-called three-factor model.

IV. The (One-Factor) CAPM and the Three-Factor Model

The theory, briefly described in the prior section, builds on *expected* returns. So how do we estimate the parameters in the model, specifically beta? The most common way is to look at historical realized data (historical returns for security j, historical risk-free rates, and historical market returns):

$$R_{j,t} - R_{F,t} = \alpha_j + \beta_j (R_{M,t} - R_{F,t}) + \varepsilon_{j,t} \tag{3}$$

$R_{j,t} - R_{F,t}$ = realized excess return on security j in period t
$R_{M,t} - R_{F,t}$ = realized excess return on the market in period t

Equation (3) is estimated using regression tools. The slope coefficient, β_j, is an estimate of the security's sensitivity to market wide movements. The model breaks down the excess return to a security into a portion attributable to market wide effects, β_j*($R_{M,t}$-$R_{F,t}$), and a portion attributable to firm-specific effects, $\alpha_j + \varepsilon_{j,t}$. Recall that firms with higher risk should have higher expected returns. The model also says that a firm's co-movement with the market return is sufficient to explain the firm's returns. Despite the presumed sufficiency of betas in explaining returns, empirical tests generally show that betas explain a relatively modest portion of the cross-sectional variation in returns.

Sometimes you will see the model estimated on returns, not excess returns (excess returns subtract the risk free rate from the firm-specific return and from the market return; returns do not adjust for the risk free rate):

$$R_{j,t} = \alpha_j + \beta_j R_{M,t} + \varepsilon_{j,t} \tag{4}$$

[1] We will not discuss the derivation, since it is purely mathematical and involves no economic arguments (it can be found in most finance textbooks).

When you estimate the model on returns rather than excess returns, you would not expect a zero intercept. Rather, the intercept now reflects the average risk-free rate (and firm-specific abnormal return) over the time period spanned by the data used to estimate the equation. In contrast, no assumption of a constant risk-free rate is necessary for estimating the models on excess returns. In the excess return version of the models, the intercept should be zero on average, unless the firm (or portfolio) has earned an abnormal return over the estimation period. For this reason, we often see people using the excess returns versions of these models to test trading rule strategies, by regressing portfolio excess returns on the risk factors and then examining the significance of the intercept α_j.

In part because of the low explanatory power of the one-factor CAPM, Fama and French [1992, 1993] explore whether other factors besides market movements explain stock returns. The other factors they find are systematically associated with security returns are firm size (market capitalization) and the ratio of the firm's book value of equity to its market value of equity ('book to market'). Based on this evidence they test the following model, the "three-factor model:"

$$R_{j,t} - R_{F,t} = \alpha_j + b_j(R_{M,t} - R_{F,t}) + s_j SMB_t + h_j HML_t + \varepsilon_{j,t} \tag{5}$$

$R_{j,t} - R_{F,t}$ = realized excess return on security j in period t,
$R_{M,,t} - R_{F,t}$ = realized excess return on the market in period t,
SMB_t = the realized return to stocks above the size breakpoint (usually defined as the NYSE median market capitalization) less the realized return to stocks below the size breakpoint, in period t,
HML_t = the realized return to stocks above a book-to-market breakpoint (usually defined as the top third of NYSE firms) less the realized return to stocks below a breakpoint (usually the bottom third of NYSE firms), in period t,
b_j, s_j, h_j = the factor loadings on the excess market portfolio, the size portfolio and the book-to-market portfolio, respectively.

Fama and French estimate the 3-factor model and find significant factor loadings (factor loadings refer to the coefficient estimates, b, s and h, on the excess market return, the SMB factor and the HML factor – so "significant factor loadings" means these variables are important in explaining security returns), high R^2s (indicating the 3-factor model explains a lot of the cross-sectional variation in security returns), and near zero alphas (suggesting market efficiency after controlling for these risk factors). Because of the widespread use of the 3-factor model, there are several databases that now report the SMB and HML returns (the market excess returns were always available, or could be easily calculated).

A. Estimating the equity cost of capital (from the one-factor CAPM)

Having estimated (or in some other way arrived at) firm j's equity beta, $\beta_{E,j}$ (same as β_j in the above notation), firm j's cost of equity capital, $r_{E,j}$, can be estimated as:

$$r_{E,j} = r_F + \beta_{E,j}[E(r_M) - r_F] \tag{6}$$

r_F = risk free rate
$E(r_M)- r_F$ = expected market risk premium[2]

[2] Six percent is advocated by Stewart [1991]. Koller, Goedhart and Wessels [2005] use 4.5%-6%. Others suggest that 7-9% is reasonable, which is how much the market has exceeded the returns on Treasury bills and Treasury

Alternatively, you may not have a series of excess returns with which to estimate the CAPM beta, or maybe you do, but you think that (for whatever reason) these returns are not representative of the firm's equity risk. Another option is to identify a set of firms that you think are comparable in terms of their equity risk, estimate the one-factor model for these firms, and use the resulting parameters to calculate the cost of equity. Using comparable firms (or the industry) to estimate a firm's beta is fairly standard as individual firm betas sometimes are unstable. We return to using comparable firms to estimate betas and cost of capital in Section D.

For the 3-factor model, costs of equity are calculated similarly, except this model requires assumptions about expected premiums for size (SMB) and book-to-market (HML) factors. Once we have those premiums, we would calculate the cost of equity as:

$$r_{E,j} = r_F + \beta_{E,j}[E(r_M) - r_F] + s_{E,j}E(SMB) + h_{E,j}E(HML)$$

B. Estimating the unlevered cost of capital

In some circumstances (like under the Adjusted Present Value (APV) approach to valuing the free cash flows to the unlevered firm), you will need to know firm *j*'s unlevered cost of capital, $r_{U,j}$. The derivation of $r_{U,j}$ follows similar to that of $r_{E,j}$.

$$r_{E,j} = r_f + \beta_{E,j}(r_m - r_f)$$
$$r_{U,j} = r_f + \beta_{U,j}(r_m - r_f)$$

where $r_{E,j}$ = firm j's cost of equity,
$r_{U,j}$ = cost of capital of the unlevered firm j,
r_f = risk free rate,
r_m = return on the market portfolio of common stocks,
$\beta_{U,j}$ = unlevered firm j beta,
$\beta_{E,j}$ = firm j's equity beta

What is an unlevered beta? – It is an unobservable measure of the co-movement of the unlevered firm's returns with market returns. It is unobservable because firms are in general *not* unlevered. The unlevered beta is intended to capture the pure operating risk of a firm (and therefore excludes any risk from financing). It is sometimes referred to as the beta of the operating assets of the firm.

Now the question is, how do you calculate unlevered betas? Since we do not observe unlevered firms, we cannot simply estimate the unlevered return regression. There are several ways around this problem; the one we will focus on is using equity and debt betas and equity and debt costs of capital to calculate unlevered betas and the unlevered cost of capital.

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bonds over the past 40-75 years. More recent research indicates that the risk premium is lower, however (in the 4% range; see, e.g., Fama and French 2002). Briefly, this research takes stock prices as given, and "backs out" the discount rate from a valuation model or a price multiple. Conversely, if you used the high historical averages (7-9%), you would under-price the entire market.

C. Levering and unlevering betas

The calculation of the firm's unlevered beta depends on what you assume about the tax shields on the firm's debt and its future capital structure plans. There are two common assumptions depending on whether debt is viewed as *fixed in dollar terms*, or *fixed as a percentage* of the capital structure (what we call a target capital structure).

Assumption 1: Interest expense is tax deductible (at a corporate tax rate = τ) and there is a fixed schedule of debt that is independent of the value of the firm. This assumption implies that the interest tax shields (ITS) should be discounted at the cost of debt. If ITS are discounted at the cost of debt and if the ITS is received in perpetuity, the present value of the ITS is just equal to the corporate tax rate times the debt:

$$\beta_U = \beta_E \left(\frac{Equity}{Equity + [(1-\tau)Debt]} \right) + \beta_D \left(\frac{(1-\tau)Debt}{Equity + [(1-\tau)Debt]} \right)$$

Note that both equity and debt should be measured in market value terms.[3] If you cannot find the market value of debt, use the fair or the book value of debt (it is often a fairly harmless simplification to assume that the book value of debt is close to the market value). In contrast, rarely is it a harmless assumption to assume that book value of equity is the same as the market value of equity, as they often differ by a factor of two or more.

Assumption 2: Interest expense is tax deductible (at a corporate tax rate = τ) and the firm wishes to maintain a constant capital structure. This assumption yields the Miles-Ezzell formulation of the unlevered beta and the unlevered cost of capital, where $q = r_D / (1 + r_D)$:

$$\beta_U = \beta_E \left(\frac{Equity}{Equity + (1-\tau q)Debt} \right) + \beta_D \left(\frac{(1-\tau q)Debt}{Equity + (1-\tau q)Debt} \right)$$

Research has shown that the majority of firms tend to have a target capital structure (explicit or implicit), so if you have no other information, Assumption 2 should be your default approach.

D. Using the unlevered and levered beta and cost of capital formulas

Some practitioners prefer not to estimate any of the above values directly for the firm they are trying to value, firm j. This is because estimating betas and costs of capital is a fairly noisy process for an individual firm.[4] Estimates are more stable when we perform the estimation on a group of firms, such as

3 If there are other types of capital in the capital structure, they are simply added to the formula. For example, if there is also preferred stock (PS) in the capital structure, the formula is:

$$\beta_U = \beta_E \left(\frac{Equity}{Equity + Pref + [(1-\tau)Debt]} \right) + \beta_{PS} \left(\frac{Pref}{Equity + Pref + [(1-\tau)Debt]} \right) + \beta_D \left(\frac{(1-\tau)Debt}{Equity + Pref + [(1-\tau)Debt]} \right)$$

4 You will get feeling for the precision of your beta estimation by inspecting the regression statistics. For example, if the explanatory power (= R-square) is very low, you may be suspicious of the quality of the estimated beta.

the firm's industry or some other set of comparable firms. For example, if we were trying to find (1) the cost of the unlevered firm j and (2) the cost of equity of firm j, we would do the following:

1. Determine the cost of equity for a set of comparable firms. For example, we estimate the CAPM, calculate the levered beta, and then infer the cost of equity for the comparable firms.

2. Using the comparable firms' cost of equity and their capital structure (together with one of the formulas noted above), we determine the cost of the unlevered comparable firm on average. This average unlevered cost can be used as an estimate of the unlevered cost of firm j.

3. Using firm j's cost of unlevered capital (just calculated), its known capital structure and information about its ITS, we can calculate firm j' cost of equity. At this point, we could then calculate firm j's weighted average cost of capital if we also knew their cost of debt.

4. Note that we do not usually estimate the cost of equity of the comparable firms and then assume this is the cost of equity for firm j. This is because the cost of equity depends on the capital structure. We take the cost of equity of the comparable firms, adjust it for the capital structure, separately for each comparable firms, to arrive at a cost of capital that is independent of capital structure (that's the cost of the unlevered firm). We then calculate the cost of equity for firm j by taking the average cost of unlevered capital from the comparables and adjust it using firm j's known capital structure. We do this because firm j's capital structure may differ from the capital structure of the comparable firms. Obviously, if the comparable firms' capital structures are very similar to that of firm j, we do not need to go through this adjustment process.

E. <u>Weighted average cost of capital</u>

For a capital structure with common equity, preferred stock, non-controlling interest,[5] and debt the weighted average cost of capital (r_{WACC}) is defined as:

$$r_{WACC} = w_E r_E + w_{PS} r_{PS} + w_{MI}(1-\tau)r_{NCI} + w_D(1-\tau)r_D$$

where r_E = cost of equity capital
r_{PS}= cost of preferred stock
r_{NCI}= cost of non-controlling interest
r_D= cost of debt capital
w_E = portion of firm's capital structure composed of common equity (measured in market value terms)
w_{PS} = portion of firm's capital structure composed of preferred stock (measured in market value terms)
w_{NCI} = portion of firm's capital structure composed of non-controlling interest (measured in market value terms)
w_D = portion of firm's capital structure composed of debt (measured in market value terms)

[5] Non-controlling interest used to be called minority interest (up until 2009). See the class note on Common and Non-Common Equity for more details.

Many times we do not know the market value of non-common-equity claims (or those market values are so noisy we are hesitant to use them). As was the case in the unlevered beta formula, it is common (but technically incorrect) to use the book value of the non-common-equity claims.

V. References

Koller, T., M. Goedhardt and D. Wessels, 2005. *Valuation: Measuring and managing the value of companies*. 4th edition. McKinsey&Co. Wiley.

Fama, E., and K. French, 1992. The cross-section of expected stock returns. *Journal of Finance* 47, 427-465.

Fama, E., and K. French, 1993. Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics* 33, 3-56.

Fama, E., and K. French, 2002. The equity premium. *Journal of Finance* 52, 637-659.